Free Surface Flows

Introduction

A number of problems of practical importance involve the low-speed flow with two-phase fluids which have large phase density ratio and a moving interface. This type of flows exists in many industrial applications. The examples can be found in the studies of droplet dynamics, capillarity, hydrodynamic stability and so on. In such flows, the fluid interfacial motion induced by surface tension plays a fundamental role. Detailed analysis of these flows involve the use of numerical models to aid in understanding the resulting nonlinear fluid flows During the last decade, general multiphase computational fluid dynamics methodologies have been proposed to simulate the flows with interfacial motions [1,2]. To calculate surface tension, the continuum surface force (CSF) model [2] is utilized. This model interprets surface tension as a continuous, three-dimensional effect across an interface, rather than as a boundary condition on the interface. The CSF method eliminates the need for interface reconstruction, and simplifies the calculation of surface tension.

Governing Equations

The current method is intended to predict the motion of fluid interfaces based on the use of a conserved scalar variable transport equation. The conserved scalar is the fractional volume of fluid (VOF) cell partitioning function, and the solution of which provides information on the position and shape of the interface. Through a linear relation, it also determines the fluid properties. The liquid phase fractional volume in a typical control volume cell is defined as:

$$\mathrm{F} = \frac{V_1}{V_g + V_1}$$

Where, V represents volumes occupied by liquid phase V1 or gas phase Vg within the control volume considered. The function F obeys the volume flux conservation equation:

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial x_i}(u_i F) = 0$$

Assuming both gas phase and liquid phase are incompressible, all other conservation equation can be written as following:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

Momentum equation:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_i} = -\frac{\partial \rho}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + g_i + F_{sv}$$

Where, p is the averaged density defined as:

$$\rho = \rho_i F + \rho_g (1 - F)$$

and , g_i is the body force(gravity). The viscous stress tensor is

$$\tau_{ij} = -\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

When the VOF equation is solved over a computational cell, the change in F in a cell reduce to fluxes of F across the cell faces. As previously noted, to calculate VOF value, we need a special numerical technique in the computation of these fluxes to preserve the sharp definition of the free boundary. In this work, a high order TVD scheme [3] with damping is used to solve the VOF value.

CSF Model for Surface Tension force

Surface tension at free surface is modeled in this work with localized volume force prescribed by the recent CSF (Continuum Surface Force) model [2]. In CSF model, instead of a surface tensile force or a surface pressure boundary condition applied as a discontinuity, a volume

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force due to surface tension on fluid elements lying within a finite thickness transition region replaces the discontinuities.

CSF formulation makes use of fact that numerical models of discontinuities in finite volume and finite difference schemes are really continuous transitions within which the fluid properties vary smoothly from one fluid to another. The volume force in CSF model is easily calculated by taking first and second order spatial derivatives of the characteristic data, which in this work is the VOF value, At each point within the free surface transition region, a cell-centered value Fgv is defined which is proportional to the curvature K of the constant VOF surface at the point. Using the formulation given in the reference [2], we have

$$\mathrm{F}_{sv} = \sigma\kappa \frac{\nabla F}{[F]}$$

Where, Q is the fluid surface tension coefficient, K is the free surface mean curvature, e which is defined as

$$\kappa = \frac{1}{|\vec{n}|}\left[\left(\frac{\vec{n}}{|\vec{n}|}\cdot\nabla\right)|\vec{n}| - (\nabla\cdot\vec{n})\right]$$

[F] is the difference of VOF data across the interface and

$$\vec{n} = \nabla F$$

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References

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